Lecture 27

Dynamic Programming: Bellman-Ford (contd.), Activity-Selection Problem

Recurrence for SSSP

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We are interested in $dist(v, n-1)$ for $v \in V(G)$

Recurrence for SSSP

Let $dist(v, i)$ = weight of a shortest path among all the paths from s to v with at most i edges.

If there is no path from s to v with at most i edges, then $dist(v, i) = \infty$

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Let $dist(v, i)$ = weight of a shortest path among all the paths from s to v with at most i edges.

Let S be the set of all the shortest paths, each of weight W, from s to v with at most i edges.

Recurrence for SSSP

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Let S be the set of all the shortest paths, each of weight W, from s to v with at most i edges.

We would like to develop a recurrence so that $dist(v, i) = W$.

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Case 1: One of the paths in S has at most $i - 1$ edges.

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Let $dist(v, i)$ = weight of a shortest path among all the paths from s to v with at most i edges.

Let S be the set of all the shortest paths, each of weight W, from s to v with at most i edges.

Case 1: One of the paths in S has at most $i - 1$ edges.

Case 2: All the paths in S have exactly i edges.

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Case 1: One of the paths in S has at most $i - 1$ edges. Then,

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Case 1: One of the paths in S has at most $i - 1$ edges. Then,

$$W = dist(v, i - 1)$$

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Let S be the set of all the shortest paths, each of weight W, from s to v with at most i edges.

Case 1**:** One of the paths in S has at most $i - 1$ edges. Then,

$$W = dist(v, i - 1)$$

↑

Clearly, $dist(v, i - 1) > W$ is not possible.

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Case 1**:** One of the paths in S has at most $i - 1$ edges. Then,

$$W = dist(v, i - 1)$$

↑

$dist(v, i - 1) < W$ is also not possible.

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Let $dist(v, i)$ = weight of a shortest path among all the paths from s to v with at most i edges.

Let S be the set of all the shortest paths, each of weight W, from s to v with at most i edges.

Case 1**:** One of the paths in S has at most $i - 1$ edges. Then,

$$W = dist(v, i - 1)$$

$dist(v, i - 1) < W$ is also not possible. Otherwise, it will contradict.

Recurrence for SSSP

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Let S be the set of all the shortest paths, each of weight W, from s to v with at most i edges.

Case 1**:** One of the paths in S has at most $i - 1$ edges. Then,

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Let $dist(v, i)$ = weight of a shortest path among all the paths from s to v with at most i edges.

Let S be the set of all the shortest paths, each of weight W, from s to v with at most i edges.

Case 2: All the paths in S have exactly i edges.

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Let $dist(v, i)$ = weight of a shortest path among all the paths from s to v with at most i edges.

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Case 2: All the paths in S have exactly i edges. Let P be one of the paths in S.

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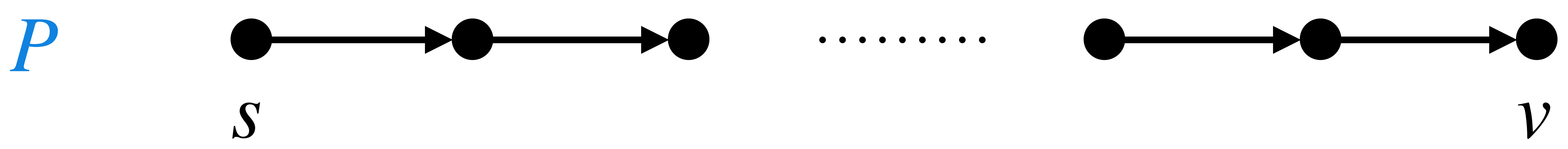

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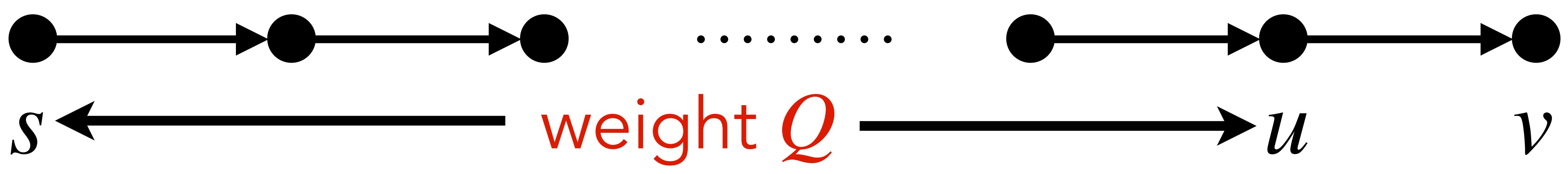

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$$W = Q + w(u, v)$$

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$Q = dist(u, i-1)$, $\because$ a shorter path from s to u with at most $i-1$ edges will contradict

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$$dist(v, i) = \begin{cases} 0, & \text{if } i = 0 \text{ and } v = s \\ & \\ & \end{cases}$$

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Why this will not be less than W?

Bellman-Ford: Bottom-Up DP for SSSP

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SSSP(G, s)

Bellman-Ford: Bottom-Up DP for SSSP

SSSP(G, s)

1. $d[1:n],\ dist[1:n][0:n-1] = \{\infty, \infty, \ldots, \infty\}$

Bellman-Ford: Bottom-Up DP for SSSP

$n = |V(G)|$

SSSP(G, s)

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Bellman-Ford: Bottom-Up DP for SSSP

SSSP(G, s)

1. $d[1:n],\ dist[1:n][0:n-1] = \{\infty, \infty, \ldots, \infty\}$

$dist[v][i]$ stores weight of a shortest path from s to v using at most i edges.

Bellman-Ford: Bottom-Up DP for SSSP

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SSSP(G, s)

1. $d[1:n],\ dist[1:n][0:n-1] = \{\infty, \infty, \ldots, \infty\}$
2. $dist[s][0] = 0$

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1. $d[1:n],\ dist[1:n][0:n-1] = \{\infty, \infty, \ldots, \infty\}$
2. $dist[s][0] = 0$
3. **for** $i = 1$ **to** $n - 1$

$dist[v][i]$ stores weight of a shortest path from s to v using at most i edges.

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3. **for** $i = 1$ **to** $n - 1$
4. **for** $v \in V(G)$

$dist[v][i]$ stores weight of a shortest path from s to v using at most i edges.

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3. **for** $i = 1$ **to** $n - 1$
4.   **for** $v \in V(G)$
5. $dist[v][i] = dist[v][i-1]$

$dist[v][i]$ stores weight of a shortest path from s to v using at most i edges.

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3. **for** $i = 1$ **to** $n - 1$
4. **for** $v \in V(G)$
5. $dist[v][i] = dist[v][i-1]$
6. **for** each edge (u, v)

$dist[v][i]$ stores weight of a shortest path from s to v using at most i edges.

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2. $dist[s][0] = 0$
3. **for** $i = 1$ **to** $n - 1$
4.   **for** $v \in V(G)$
5. $dist[v][i] = dist[v][i-1]$
6.   **for** each edge (u, v)
7. $dist[v][i] = min(dist[v][i], dist[u][i-1] + w(u, v))$

Bellman-Ford: Bottom-Up DP for SSSP

```
SSSP(G, s)
d[1 : n], dist[1 : n][0 : n − 1] = {∞, ∞, …, ∞}
dist[s][0] = 0
for i = 1 to n − 1
    for v ∈ V(G)
        dist[v][i] = dist[v][i − 1]
        for each edge (u, v)
            dist[v][i] = min(dist[v][i], dist[u][i − 1] + w(u, v))
for v ∈ V(G)
```

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4.   **for** $v \in V(G)$
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6.   **for** each edge (u, v)
7. $dist[v][i] = min(dist[v][i], dist[u][i-1] + w(u, v))$
8. **for** $v \in V(G)$
9. $d[v] = dist[v][n-1]$

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```
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        dist[v][i] = dist[v][i − 1]
            for each edge (u, v)
                dist[v][i] = min(dist[v][i], dist[u][i − 1] + w(u, v))
for v ∈ V(G)
    d[v] = dist[v][n − 1]
return d
```

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```
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$d[1:n],\ dist[1:n][0:n-1] = \{\infty, \infty, \ldots, \infty\}$
$dist[s][0] = 0$
for $i = 1$ **to** $n - 1$ ← $\Theta(|V|)$
  for $v \in V(G)$
    $dist[v][i] = dist[v][i-1]$
      for each edge (u, v)
        $dist[v][i] = min(dist[v][i], dist[u][i-1] + w(u, v))$
for $v \in V(G)$
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$dist[s][0] = 0$

for $i = 1$ **to** $n - 1$ — $\Theta(|V|)$

 for $v \in V(G)$ — $\Theta(|V| + |E|)$

  $dist[v][i] = dist[v][i-1]$

  for each edge (u, v)

   $dist[v][i] = min(dist[v][i], dist[u][i-1] + w(u, v))$

for $v \in V(G)$

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$dist[s][0] = 0$
for $i = 1$ **to** $n - 1$ — $\Theta(|V|)$
    for $v \in V(G)$ — $\Theta(|V| + |E|)$
        $dist[v][i] = dist[v][i-1]$
        for each edge (u, v) — $\Theta(|V| + |E|)$
            $dist[v][i] = min(dist[v][i], dist[u][i-1] + w(u, v))$
for $v \in V(G)$ — $\Theta(|V|)$
    $d[v] = dist[v][n-1]$
return d

Bellman-Ford: Bottom-Up DP for SSSP

```
SSSP(G, s)
d[1 : n], dist[1 : n][0 : n − 1] = {∞, ∞, …, ∞}
dist[s][0] = 0
for i = 1 to n − 1                      ← Θ(|V|)
    for v ∈ V(G)                        ← Θ(|V| + |E|)
        dist[v][i] = dist[v][i − 1]
        for each edge (u, v)            ← Θ(|V| + |E|)
            dist[v][i] = min(dist[v][i], dist[u][i − 1] + w(u, v))
for v ∈ V(G)                            ← Θ(|V|)
    d[v] = dist[v][n − 1]
return d
```

Time Complexity:

$\Theta(|V| . (|V| + |E|))$

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for $i = 1$ **to** $n - 1$

    for $v \in V(G)$

        $dist[v][i] = dist[v][i-1]$

        for each edge (u, v)

            $dist[v][i] = min(dist[v][i], dist[u][i-1] + w(u, v))$

for $v \in V(G)$

    $d[v] = dist[v][n-1]$

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Time Complexity:

$\Theta(|V| . (|V| + |E|))$

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2. $dist[s][0] = 0$
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4.   **for** $v \in V(G)$
5. $dist[v][i] = dist[v][i-1]$
6.   **for** each edge (u, v)
7. $dist[v][i] = min(dist[v][i], dist[u][i-1] + w(u, v))$
8. **for** $v \in V(G)$
9. $d[v] = dist[v][n-1]$
10. **return** d

Modify line 4 to 7 so that this cost becomes only $\Theta(|E|)$.

Time Complexity:

$\Theta(|V| . (|V| + |E|))$

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Two activities a_i and a_j are mutually compatible if either $s_i \geq f_j$ or $s_j \geq f_i$.

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Example:

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s_i											
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